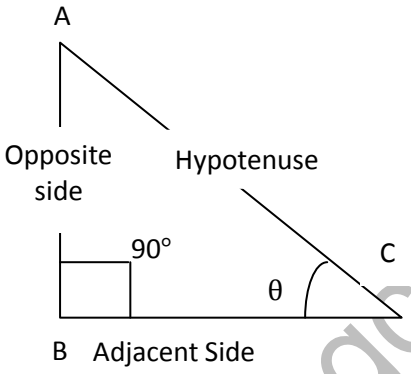
TRIGONOMETRY

❖ **Trigonometrical functions (ratios)** : In a right angled triangle the relations between the sides and angles are trigonometric ratios. They are “sine”, “cosine”, “tangent”. “cosecant”, “secant” and “cotangent”.

1. $\sin\theta = \frac{\text{opposite side to } \theta}{\text{Hypotenuse}} = \frac{AB}{AC}$
2. $\cos\theta = \frac{\text{Adjacent side to } \theta}{\text{Hypotenuse}} = \frac{BC}{AC}$
3. $\tan\theta = \frac{\text{opposite side to } \theta}{\text{Adjacent side to } \theta} = \frac{AB}{BC}$
4. $cosec\,\theta = \frac{\text{Hypotenuse}}{\text{opposite side to } \theta} = \frac{AC}{AB}$
5. $\sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent side to } \theta} = \frac{AC}{BC}$



❖ $\cot\theta = \frac{\text{Adjacent side to } \theta}{\text{opposite side to } \theta} = \frac{BC}{AB}$

❖ **Relation between trigonometric Ratio’s :**

1. $\text{cosec}\,\theta = \frac{1}{\sin\theta}$ or $\sin\theta = \frac{1}{\text{cosec }\theta}$
2. $\sec\theta = \frac{1}{cos\theta}$ or $\cos\theta = \frac{1}{\sec\theta}$
3. $\cot\theta = \frac{1}{\cot\theta}$ or $\tan\theta = \frac{1}{\cot\theta}$
4. $\tan\theta = \frac{\cos\theta}{\sin\theta}$ or $\cot\theta = \frac{\cos\theta}{\sin\theta}$
5. $\sin\theta.cosec\,\theta = 1,\ \cos\theta.\sec\theta = 1,\ \tan\theta.\cot\theta = 1,$

❖ The values of trigonometric ratios for various angles:

θ	$0°$	$30°$	$45°$	$60°$	$90°$
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$cosec\ \theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot\theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

❖ The values of $\sin\theta$ and $\cos\theta$ always lie between 0 and 1.

❖ In case of $\tan\theta$ the values increases from 0 to ∞.

❖ In case of $\cot\theta$, the values decreases from ∞ to 0.

❖ In case of $cosec\ \theta$, the values devreases from ∞ to 1.

❖ In case of $\sec\theta$, the values increases from 1 to ∞.

❖ Trigonometric Identities:

1) $\sin^2\theta + \cos^2\theta = 1$
2) $\sec^2\theta - \tan^2\theta = 1$
3) $\text{cosec}^2\theta - \cot^2\theta = 1$

❖ Fundamental trigonometrical identities :

a) $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$

$\Rightarrow \sin\theta = \sqrt{1 - \cos^2\theta}$

$\Rightarrow \cos^2\theta = 1 - \sin^2\theta$

$\Rightarrow \cos\theta = \sqrt{1 - \sin^2\theta}$

b) $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow \sec\theta = \sqrt{1 + \tan^2\theta}$

$\tan^2\theta = \sec^2\theta - 1$

$\Rightarrow \tan\theta = \sqrt{\sec^2\theta - 1}$

$sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$

$\text{Sec}\sec\theta - \tan\theta = \frac{1}{\sec\theta = \tan\theta}$

c) $\text{cosec}^2 - \cot^2\theta = 1$

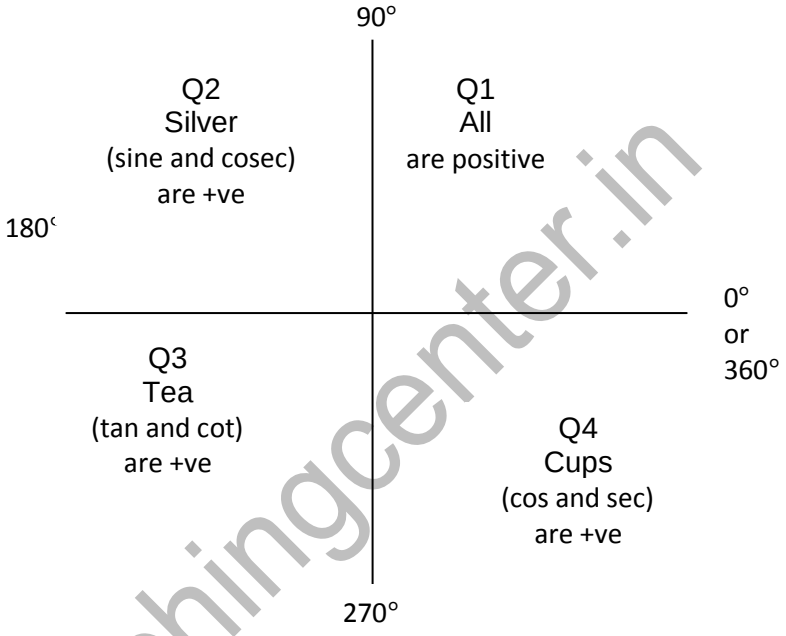
$\Rightarrow \text{cosec}^2\theta = 1 + \cot^2\theta$

$\Rightarrow cosec\,\theta = \sqrt{1 + \cot^2\theta}$

$\Rightarrow \cot^2\theta = \text{cosec}^2\theta - 1$

$\Rightarrow \cot\theta = \sqrt{cosec^2\theta - 1}$

$(cosec\,\theta + \cot\theta) = \frac{1}{\text{cosec}\,\theta - \cot\theta}$



$\sin(90-\theta) = cos\theta$	$\sin(180-\theta) = sin\theta$
$\sin(90+\theta) = cos\theta$	$\sin(180+\theta) = -sin\theta$
$\cos(90-\theta) = sin\theta$	$\cos(180-\theta) = -cos\theta$
$\cos(90+\theta) = -sin\theta$ (since cos is –ve in Q2)	$\cos(180+\theta) = -cos\theta$
$\tan(90-\theta) = cot\theta$	$\tan(180-\theta) = -tan\theta$
$\tan(90+\theta) = -cot\theta$ (since tan is –ve in Q2)	$\tan(180+\theta) = tan\theta$
$\text{cosec}(90-\theta) = sec\theta$	$\text{cosec}(180-\theta) = cosec\theta$
$\text{cosec}(90+\theta) = sec\theta$	$\text{cosec}(180+\theta) = -cosec\theta$
$\sec(90-\theta) = cosec\theta$	$\sec(180-\theta) = -sec\theta$
$\sec(90+\theta) = -cosec\theta$	$\sec(180+\theta) = -sec\theta$
$\cot(90-\theta) = tan\theta$	$\cot(180-\theta) = -cot\theta$
$\cot(90+\theta) = -tan\theta$ (since cot is –ve in Q2)	$\cot(180+\theta) = cot\theta$
$\sin(270-\theta) = -cos\theta$	$\sin(360-\theta) = -sin\theta$
$\sin(270+\theta) = -cos\theta$	$\sin(360+\theta) = sin\theta$
$\cos(270-\theta) = -sin\theta$	$\cos(360-\theta) = cos\theta$
$\cos(270+\theta) = sin\theta$	$\cos(360+\theta) = cos\theta$
$\tan(270-\theta) = cot\theta$	$\tan(360-\theta) = -tan\theta$
$\tan(270+\theta) = -cot\theta$ (since tan is –ve in Q4)	$\tan(360+\theta) = tan\theta$
$\text{cosec}(270-\theta) = -sec\theta$	$\text{cosec}(360-\theta) = -cosec\theta$
$\text{cosec}(270+\theta) = -sec\theta$	$\text{cosec}(360+\theta) = cosec\theta$
$\sec(270-\theta) = -cosec\theta$	$\sec(360-\theta) = sec\theta$
$\sec(270+\theta) = cosec\theta$	$\sec(360+\theta) = sec\theta$
$\cot(270-\theta) = tan\theta$	$\cot(360-\theta) = -cot\theta$
$\cot(270+\theta) = -tan\theta$ (since cot is –ve in Q4)	$\cot(360+\theta) = cot\theta$

❖ **Compound angles :** The algebraic sum of two or more angles is called compound angles.

i) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

ii) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

iii) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

iv) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

v) $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

vi) $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

vii) $\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$

viii) $\cot(A-B) = \frac{\cot A \cot B + 1}{\cot A - \cot B}$

❖ **Multiple and Sub-multiple angles :**

If 'A' is any angle then the angles 2A, 3A,4A etc., are said to be multiple angles and the angles $\frac{A}{2}, \frac{3A}{2}, \frac{A}{3}, \frac{A}{4}$ etc., are said to be sub-multiple angles.

i) $\sin 2A = 2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A}$

ii) $\sin A = 2 \sin \frac{A}{2} \cos \frac{A}{2} = \frac{2 \tan \frac{A}{2}}{1 - \tan^2 \frac{A}{2}}$

iii) $\cos 2A = cos^2 A - sin^2 A = 2\, cos^2 A - 1 = 1 - 2\, sin^2 A = \frac{1 - tan^2 A}{1 + tan^2 A}$

iv) $\sin 3A = 3 \sin A - 4 \sin^3 A$

v) $\cos 3A = 4 \cos^3 A - 3 \cos A$

vi) $\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$

vii) $\cos A = \cos^2 \frac{A}{2} - sin^2 \frac{A}{2} = 2\, cos^2 \frac{A}{2} - 1 = 1 - 2\, sin^2 \frac{A}{2} = \frac{1 - tan^2 \frac{A}{2}}{1 + tan^2 \frac{A}{2}}$

viii) $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

ix) $\tan A = \frac{2 \tan \frac{A}{2}}{1 - \tan^2 \frac{A}{2}}$